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TOP QUARK CONDENSATES

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ABSTRACT

We describe the top condensate scheme for electroweak symmetry breaking, and some associated issues. We discuss the issue of predictability, and elaborate a "topcolor" model for a new gauge interaction that can drive the formation of the $\bar{t}t$ condensate. We find corrections to the naive Nambu–Jona-Lasinio approximation to be small.

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1. Introduction

Many physicists believe that the true symmetry breaking of the electroweak interactions involves a dynamical mechanism in analogy to the BCS theory. The most celebrated mechanism is that of technicolor[1] and involves a new strong interaction that pairs techniquarks to form a weak isospin–1/2 condensate. This is, in the earliest version, a well understood dynamical mechanism for electroweak symmetry breaking, by analogy to the well–known chiral symmetry breaking in QCD, but it does not account for the masses of quarks and leptons. To give masses to the elementary fermions one must extend technicolor to a larger, broken symmetry group, known as "extended technicolor"[2] and here one encounters difficulties. The key problem is that quarks cannot be too heavy in extended technicolor schemes without simultaneously generating large, unwanted $\Delta S = 2$ interactions, thus too large a $K_L - K_S$ mass difference. A naive estimate of the upper limit on a quark mass in standard extended technicolor is $m_{quark} \lesssim 100$ MeV. In a more recent version, "walking extended technicolor,"[3] one might obtain larger masses, $m_{quark} \lesssim 10$ GeV.

The fact that the top quark is an elementary fermion with a mass scale of order the electroweak symmetry breaking scale suggests a dramatic new possibility: *the symmetry breakdown of the standard model may be a dynamical mechanism which intimately involves the top quark.* To implement this idea we will postulate new dynamics in which the top quark forms a condensate, for example $\langle \bar{t}_L t_R + h.c. \rangle$, which has the correct electroweak isospin–$\frac{1}{2}$ quantum number. Thus, in this scheme the top quark itself plays the role of a techniquark.

There is clearly much uncertainty in the specific new dynamics leading to top quark condensation. As a first step toward a full theory one can implement directly a Nambu–Jona-Lasinio (NJL) mechanism in which an effective $SU(3) \times SU(2) \times U(1)$ invariant four–fermion interaction associated with a high energy scale, $\Lambda \sim G^{-1/2}$, is postulated[4,5]:

$$\mathcal{L} = \mathcal{L}_{kinetic} + G(\overline{\Psi}_L^{ia} t_{Ra})(\bar{t}_R^b \Psi_{Lib}) \tag{1}$$

where i runs over $SU(2)_L$ indices, (a, b) run over color indices, and $\mathcal{L}_{kinetic}$ contains the usual gauge invariant fermion and gauge boson kinetic terms. There is no elementary Higgs field in $\mathcal{L}$. If $G > 0$ the interaction is attractive, and for sufficiently large G the four–fermion interaction triggers the formation of a low energy condensate, $\langle \bar{t}t \rangle$, which breaks $SU(2) \times U(1) \to U(1)$.

The bootstrapping of the symmetry breaking mechanism to the top quark produces the requisite Nambu–Goldstone bosons associated with spontaneous symmetry breaking (which ultimately become the longitudinal components of W and Z), and also a composite particle which behaves identically to a fundamental Higgs boson at low energies.

By virtue of its economy this theory leads to new predictions which are testable in the near future. In particular, we are able to derive renormalization group improved predictions for m_{top} and m_{Higgs} (the composite $\bar{t}t$ Higgs boson) in this scheme, and we find, not surprisingly, that m_{top} is of order the weak scale. The results are very weakly dependent upon Λ; for example, with $\Lambda \sim 10^{15}$ GeV we find in the minimal scheme $m_{top} \approx 230$ GeV and $m_{Higgs} \sim 260$ GeV.[5] Yet another result, albeit not experimentaly accessible in the foreseeable future, is that the nonminimal coupling of the composite Higgs boson to gravity is determined, and we find the conformal value, $\xi = 1/6$ as a general consequence of compositeness in the NJL model.[6]

Thus, this model differs from technicolor at the outset in implying that at least one fermion, that associated with the electroweak condensate, must be heavy while the others are light. The usual Cabibbo–Kobayashi–Maskawa mixing angle structure and light fermion mass spectrum are readily accomodated, but predictions of mixing angles and the light quark masses are not derivable until one specifies the dynamics at the scale Λ more precisely. The usual one–Higgs–doublet standard model emerges as the low energy effective Lagrangian, but with new constraints that lead to the nontrivial predictions for m_{top}, m_{Higgs} and ξ.

The NJL model is conventionally treated in a large N_{color} approximation, keeping only the effects of fermion loops. However, one can equivalently analyze the model using the renormalization group (RG) exclusively. This involves studying the effective Lagrangian and the evolution of its parameters as we vary the scale of physics, μ. At the high energies, $\mu \sim G^{-1/2}$ our theory is described by the four–fermion interaction of eq.(1). At low energies it contains a dynamical, composite weak isodoublet Higgs boson with self interactions and a Higgs–Yukawa coupling to the top quark. We must then understand how to "match" the low energy Lagrangian onto the high energy Lagrangian. The conditions that define this matching are called the "compositeness conditions."[5,7] The compositeness conditions are equivalent to boundary conditions near the scale Λ on the renormalization group equations. With the correct compositeness conditions we easily recover the conventional NJL results in the large–N limit.

The compositeness conditions are actually more powerful; they may be applied to *the full theory*, which goes beyond the large–N approximation and includes the effects of gauge boson and internal Higgs boson lines, *etc.* Certain special renormalization group trajectories, *i.e.*, those satisfying the compositeness conditions, are thereby associated with the existence of composite structure. These lead to the precise RG improved predictions for m_{top}, m_{Higgs}, and ξ, which are very insensitive to the scale of new physics, $\Lambda \sim G^{-1/2}$.[5]

The composite theory is effectively a strongly coupled (Higgs-Yukawa and quartic Higgs couplings) standard model near the scale Λ. The low energy predictions that emerge are found to be governed in each case by infrared renormalization group fixed points.[8,9] In particular, compositeness is associated with the infrared fixed points as formulated in ref.[9]. These fixed points are universal low energy values of the coupling constants that arise from arbitrarily large values at high energies. Because the low energy values are insensitive to a wide range of initial values, the compositeness predictions are robust, and largely insensitive to the precise details of the high energy theory. For example, the top quark is predicted to lie near 230 GeV with the Higgs near 260 GeV, and $\xi = 1/6$ for a composite scale within several orders of magnitude of, $\Lambda \sim 10^{15}$ GeV.

How robust are the compositeness conditions and hence the predictions of a theory as in eq.(1)? One can follow Suzuki[10] and consider the sensitivity of the results to the presence of generic higher dimension operators. Again, owing to the infrared fixed points, the results are found to be very insensitive to higher dimension operators (in ref.[11] arbitrarily large coefficients of these operators are allowed and it is claimed that the infrared predictions of the composite theory can be modified; we will return to this issue in section 4.2). It is important to realize, however, that the theory of eq.(1) cannot be viewed as fundamental. One is therefore challenged to construct models in which eq.(1) emerges as the effective theory at the scale Λ. With a wide class of such models we can compute the strength of irrelevant operators.

To get a sense of the expected size of these additional effects in a realistic scheme, and to have a concrete realization (albeit not necessarily the most elegant), we will give a discussion of a "topcolor" model in analogy to minimal technicolor.[12] While minimal technicolor is a theory which naturally breaks $SU(2) \times U(1)$ but leaves the fermions massless, minimal topcolor breaks the electroweak interactions with a dynamical top condensate, while leaving all other quarks and leptons massless. This may be a better point of departure for the construction of extended models in which all quarks and leptons receive masses, however this is a new subject and we will not pursue the development of detailed schemes in this paper.

In the end we face the fundamental problem of "naturalness," *i.e.*, how to evade significant fine-tuning of the theory. Two avenues have evolved: (1) SUSY generalizations of the minimal model[13] in which supersymmetry protects the gap equation from having to fine-tune large quadratic divergences; and (2) Fourth generation schemes[14] in which the scale Λ is simply taken near ~ 1 TeV. Unfortunately, the limitations of space will preclude a review of these, and we refer the reader to the references.

2. Analysis in Fermion Bubble Approximation

We can see explicitly the connection with the standard model of the theory defined in eq.(1) by using a Yukawa form of the four–fermion interactions as defined at the cut–off scale Λ, through the help of a static, auxiliary Higgs field, H. We can rewrite eq.(1) as:

$$\mathcal{L} = \mathcal{L}_{kinetic} + (\overline{\Psi}_L t_R H + h.c.) - m_0^2 H^\dagger H \tag{2}$$

If we integrate out the field H we produce the four–fermion vertex as an induced interaction with $G = 1/m_0^2$. Note that only nontachyonic $m_0^2 > 0$ implies an attractive interaction and allows the factorization in this form.

Eq.(2) is the effective Lagrangian on a scale Λ. To obtain the effective Lagrangian on a scale $\mu < \Lambda$ in the fermion bubble approximation we integrate out the fermion field components on scale $\mu \to \Lambda$ as in Fig.(1):

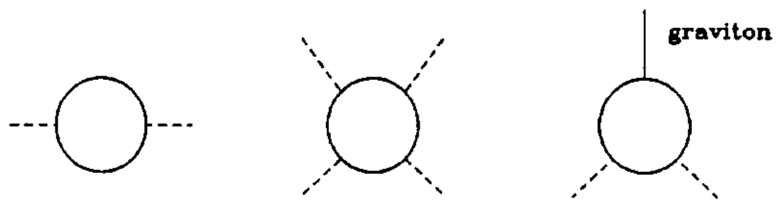

Figure (1): Block–spin renormalization group including only fermion loops.

The full induced effective Lagrangian at the scale μ then takes the form:

$$\begin{aligned} \mathcal{L} &= \mathcal{L}_{kinetic} + \overline{\Psi}_L t_R H + h.c. + \Delta\mathcal{L}_{gauge} \\ &\quad + Z_H|D_\nu H|^2 - m_H^2 H^\dagger H - \frac{\lambda_0}{2}(H^\dagger H)^2 - \xi_0 R H^\dagger H \end{aligned} \tag{3}$$

where D_ν is the gauge covariant derivative and all loops are now to be defined with respect to a low energy scale μ. Here $\Delta\mathcal{L}_{gauge}$ contains the fermion loop contribution to the renormalization of the gauge coupling constants. We include an induced non-minimal coupling of the Higgs boson to gravity, ξ.[6] A direct evaluation of the induced parameters in the Lagrangian gives as in Fig.(1):

$$\begin{aligned} Z_H &= \frac{N_c}{(4\pi)^2}\log(\Lambda^2/\mu^2); \qquad m_H^2 = m_0^2 - \frac{2N_c}{(4\pi)^2}(\Lambda^2-\mu^2) \\ \lambda_0 &= \frac{2N_c}{(4\pi)^2}\log(\Lambda^2/\mu^2); \qquad \xi_0 = \frac{1}{6}\frac{N_c}{(4\pi)^2}\log(\Lambda^2/\mu^2). \end{aligned} \tag{4}$$

The Lagrangian of eq.(3) is, apart from normalization, exactly the same as the usual low energy standard model, except that the induced parameters, Z_H and λ_0, and ξ_0 are determined. Note that they all have an explicit dependence upon Λ, vanishing when $\mu \to \Lambda$.

We emphasize that *the effective theory applies in either the broken or unbroken phases.* The broken phase is selected by demanding that $m_H^2 < 0$ for scales $\mu \ll \Lambda$, thus requiring that $m_0^2 - 2N_c\Lambda^2/16\pi^2 < 0$. *This is equivalent to tuning the gap equation to produce the low energy dynamical symmetry breaking, i.e.,* $G > G_c = 8\pi^2/N_c\Lambda^2$ since $G = 1/m_0^2$. On the other hand, for positive m_H^2 as $\mu \to 0$ the theory remains unbroken (this is equivalent to a subcritical four-fermion coupling constant, $G \le G_c$) and a massive Higgs boson doublet remains in the spectrum as a composite state.

Let us bring the effective Lagrangian of eq.(3) into a conventionally normalized form:

$$\begin{aligned} \mathcal{L} &= \mathcal{L}_{kinetic} + g_t\overline{\Psi}_L t_R H + h.c. + \Delta\mathcal{L}_{gauge} \\ &\quad + |D_\nu H|^2 - \widetilde{m}_H^2 H^\dagger H - \frac{\lambda}{2}(H^\dagger H)^2 - \xi R H^\dagger H \end{aligned} \tag{5}$$

by rescaling the field $H \to H/\sqrt{Z_H}$. We then find:

$$\begin{aligned} g_t^2 &= 1/Z_H = 16\pi^2/N_c\log(\Lambda^2/\mu^2) \\ \widetilde{m}_H^2 &= m_H^2/Z_H \\ \lambda &= \lambda_0/Z_H^2 = 32\pi^2/N_c\log(\Lambda^2/\mu^2) \\ \xi &= \xi_0/Z_H = 1/6 \end{aligned} \tag{6}$$

These are the physically normalized coupling constants, and after fine-tuning the low energy value of $\widetilde{m}_H^2$ to obtain the spontaneously broken phase, the remaining predictions of the model are contained entirely in g_t, λ (and ξ) as we will see below.

The compositeness of the Higgs boson essentially implies the results that g_t and λ become singular as $\mu \to \Lambda$ (while ξ remains constant and equal to its conformal value of 1/6). We will refer to these as the "compositeness conditions."

Now, to obtain the usual phenomenological results of the NJL model we examine the low energy Higgs potential from eq.(5) with $\mu = m_t$:

$$V(H) = -\widetilde{m}_H^2 H^\dagger H + \frac{\lambda}{2}(H^\dagger H)^2 - (g_t \overline{\Psi}_L t_R H + h.c.) \tag{7}$$

Let us assume that $\widetilde{m}_H^2 < 0$ so the neutral Higgs field develops a VEV: $Re(H^0) = v + \phi/\sqrt{2}$. In the standard model we assume that v has been fine–tuned to the physical value of $v^2 = 1/2\sqrt{2}G_F = (175)^2$ GeV.

Therefore we find the top mass:

$$m_t = g_t v; \tag{8}$$

and the ϕ mass:

$$m_\phi^2 = 2v^2\lambda \tag{9}$$

and so:

$$m_\phi^2/m_t^2 = 2\lambda/g_t^2 = 4 \tag{10}$$

where we use the explicit results of eq.(6) for $\lambda/g_t^2 = 2$. This is the familar NJL result, $m_\phi = 2m_t$. Moreover, we have:

$$v^2 = m_t^2/g_t^2 = m_t^2 \frac{N_c}{16\pi^2}\ln(\Lambda^2/m_t^2) = \frac{1}{2\sqrt{2}G_F} \tag{11}$$

which is equivalent to the prediction obtained from a direct fermion bubble approximation computation of the decay constant. We have seen that the RG directly and simply reproduces the result of a "brute force" summation of fermion bubbles. The result $\xi = 1/6$ is also seen readily in the differential renormalization group.[6]

3. Fully Improved Renormalization Group Solution

3.1 Infrared Fixed Points

To obtain the full renormalization group improvement over the Nambu–Jona-Lasinio model we may utilize the compositeness boundary conditions on g_t and λ and the *full* β-functions (we'll neglect light quark masses and mixings) of the standard model. To one–loop order we have:

$$16\pi^2 \frac{\partial}{\partial \ln \mu} g_t = \left((N_c + \frac{3}{2}) {g_t}^2 - (N_c^2 - 1) {g_3}^2 - \frac{9}{4} {g_2}^2 - \frac{17}{12} {g_1}^2 \right) g_t \tag{12}$$

and, for the gauge couplings:

$$16\pi^2 \frac{\partial}{\partial \ln \mu} g_i = -c_i \; {g_i}^3 \tag{13}$$

with

$$c_1 = -\frac{1}{6} - \frac{20}{9} N_g; \quad c_2 = \frac{43}{6} - \frac{4}{3} N_g; \quad c_3 = 11 - \frac{4}{3} N_g \tag{14}$$

where N_g is the number of generations and $t = \ln \mu$.

The precise value of the top quark mass is determined by running $g_t(\mu)$ down from a given compositeness scale Λ at which $g_t(\Lambda) = \infty$, or in practice, is large. The evolution ends when the mass–shell condition $g_t(m_t)v = m_t$ is reached. We will not discuss possible low energy corrections associated with the extrapolation of the symmetric three–point function to a zero–momentum Higgs line.

The nonlinearity of eq.(12) focuses a wide range of initial values into a small range of final low energy results.[8,9] The solution for $m_{quark} = g_t(\mu)v$ is shown in Fig.(5) for $\Lambda = 10^{15}$ GeV (case A) and $\Lambda = 10^{19}$ GeV (case B) respectively. This is a "quasi" infrared fixed point, which would be an exact fixed point if g_3 were constant. The fixed point is a reflection of approximate scale invariance (vanishing β function) of the theory as we tune the gap equation to produce $m_t \ll \Lambda$. The scale invariance is explicitly broken by Λ_{QCD}.

The quasi–fixed point behavior implies that m_t is determined up to $O(\ln \ln \Lambda / m_t)$ sensitivity to Λ. In Table I we give the resulting physical m_{top} obtained by a numerical solution of the renormalization group equations as a function of Λ. Note the sensitivity to Λ is reduced when the nontrivial IR fixed point is present.

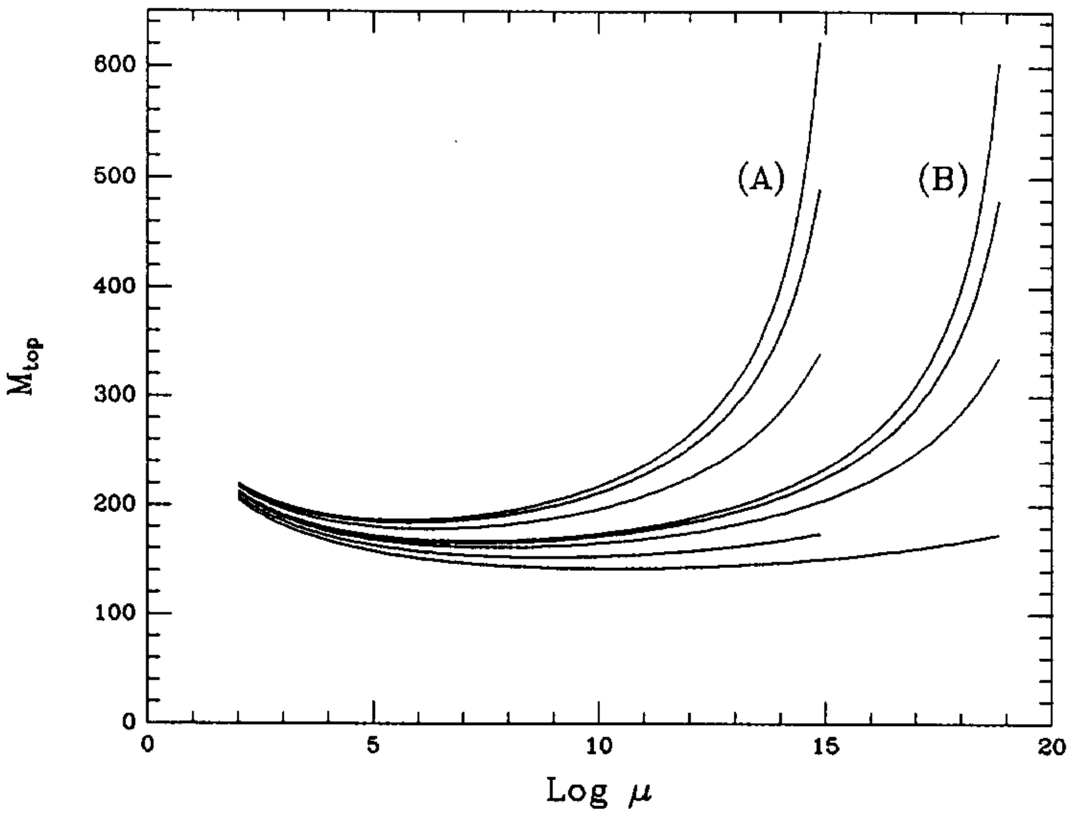

Figure (2): Full RG trajectories as a function of scale μ. (A) $\Lambda = 10^{15}$ GeV; (B) $\Lambda = 10^{19}$ GeV. The composite trajectories diverge at the corresponding value of Λ. The predicted m_{quark} is controlled by the quasi-infrared fixed point and is very insensitive to Λ.[5,9]

The Higgs boson mass will likewise be determined by the evolution of λ now given by:

$$16\pi^2 \frac{\partial}{\partial \ln \mu}\lambda = 12(\lambda^2 + (g_t{}^2 - A)\lambda + B - g_t{}^4) \tag{15}$$

where:

$$A = \frac{1}{4}g_1{}^2 + \frac{3}{4}g_2{}^2; \quad B = \frac{1}{16}g_1{}^4 + \frac{1}{8}g_1{}^2 g_2{}^2 + \frac{3}{16}g_2{}^4 \tag{16}$$

As in the case of g_t, we evolve $\lambda(\mu)$ from the compositeness scale Λ down to the weak scale with the compositeness boundary condition, $\lambda(\mu \to \Lambda) \to \infty$. The joint evolution of g_t and λ to the RG fixed point is shown in Fig.(3), and m_H is given in Table I including the full RG effects.

Λ (GeV)	10^{19}	10^{15}	10^{11}	10^{7}	10^{5}
m_t (GeV); Fermion Bubble[a]	144	165	200	277	380
m_t (GeV); Planar QCD[a]	245	262	288	349	432
m_t (GeV); Full RG[b]	218	229	248	293	360
m_H (GeV); Full RG[b]	239	256	285	354	455

Table I: Predicted m_{top} in three levels of increasingly better approximation as described in the text. "Fermion Bubble" refers only to the inclusion of fermion loops, equivalent to the conventional Nambu–Jona-Lasinio analysis, in which case $m_H = 2m_t$. "Planar QCD" includes additional effects of internal gluon lines. All effects, including internal Higgs lines and electroweak corrections, are incorporated in the "Full RG" lines, and we include the m_H results. *Notice that the full renormalization effects cause $m_H \neq 2m_t$.* Results ([a]) are from Mahanta and Barrios,[15] and ([b]) are from ref.[5].

3.2 Sensitivity to Irrelevant Operators

The action of the effective fixed point appears to make the top quark and Higgs boson mass predictions largely insensitive to the precise values of the coupling constant close to the scale Λ.[8,9] Indeed, there may be real physical effects which modify the high energy boundary conditions. These effects may be due to the presence of normally irrelevant, higher dimension operators, or higher order corrections to the four fermion interactions at the scale Λ which are not already contained in the full renormalization group analysis. The higher dimension operators were first considered by Suzuki[10] and his analysis has been generalized by Hasenfratz[11] *et al.* How sensitive is the infrared physics to these model-dependent effects at high energy? We will show that these "Suzuki effects" are in fact rather small for a reasonable range of the coefficients of these new operators.

We take our starting point Lagrangian, eq.(1), to be modified as

$$L = L_{kinetic} + G\left(\overline{\Psi}_L^{ia} t_{Ra} + \frac{\chi}{\Lambda^2}(D_\mu \overline{\Psi}_L^{ia})(D^\mu t_{Ra})\right)\left(\bar{t}_R^b \Psi_{Lib} + \frac{\chi}{\Lambda^2}(D_\mu \bar{t}_R^b)(D^\mu \Psi_{ibL})\right) \quad (17)$$

hence eq.(2) is similarly modified:

$$L = L_{kinetic} + \left((\overline{\Psi}_L^{ia} t_{Ra} + \frac{\chi}{\Lambda^2}(D_\mu \overline{\Psi}_L^{ia})(D^\mu t_{Ra}))H_i + h.c. \right) - M_0^2 H^\dagger H \qquad (18)$$

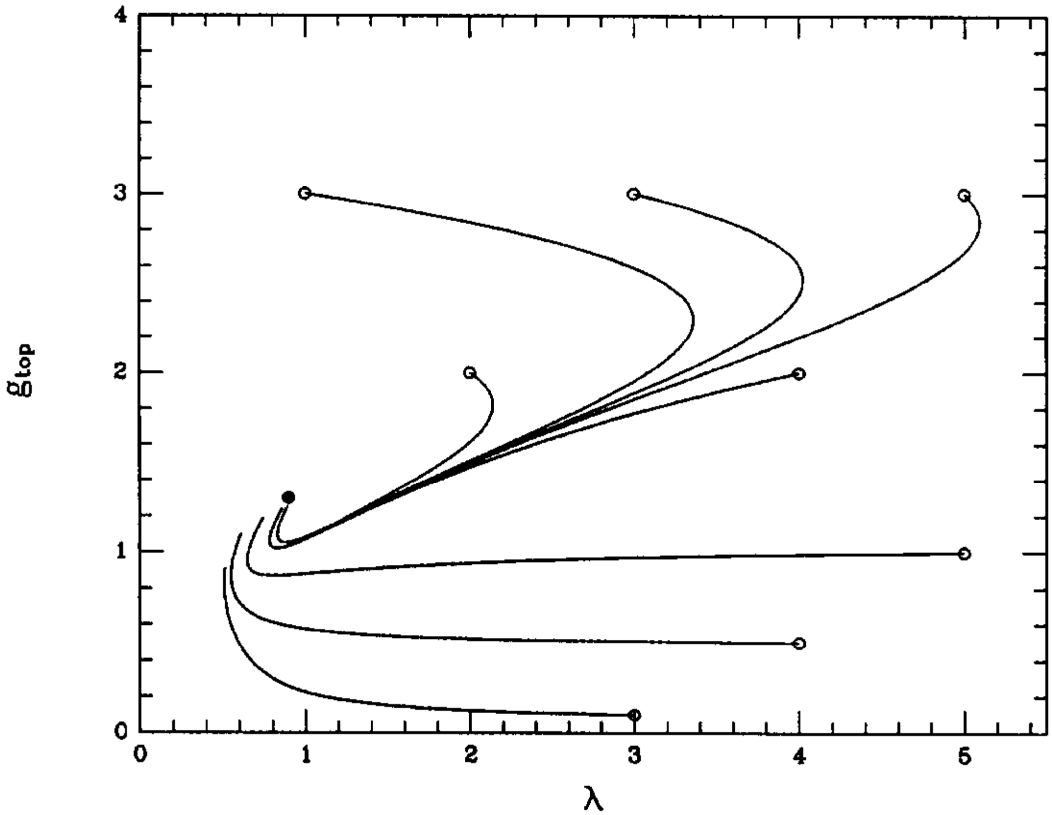

Figure (3): Full RG trajectories showing joint evolution of g_t and λ for various initial values.[9] Compositeness corresponds to large initial g_t and λ, and these are attracted toward the nontrivial IR fixed point (solid circle).

Now, we perform the block-spin RG transformation as in section 2.1. we obtain the low energy effective Lagrangian in analogy with eq.(3):

$$\begin{aligned} L &= L_{kinetic} + \left((\overline{\Psi}_L^{ia} t_{Ra} H_i + h.c. \right) \\ &\quad + Z_H |D_\mu H|^2 - M_\mu^2 H^\dagger H - \frac{\lambda_0}{2}(H^\dagger H)^2 + O(1/\Lambda^2)... \end{aligned} \qquad (19)$$

where now the parameters transform as:

$$Z_H = \frac{N_c}{8\pi^2}\left(\ln(\Lambda/\mu) - \chi + \chi^2/8\right) \tag{20}$$

$$\lambda_0 = \frac{N_c}{4\pi^2}\left(\ln(\Lambda/\mu) - 2\chi + \frac{3}{2}\chi^2 - \frac{2}{3}\chi^3 + \frac{1}{8}\chi^4\right) \tag{21}$$

and M_μ^2 has additive terms which we will fine–tune as above.

To obtain the low energy predictions the renormalizaton group equations are modifeid by physics near the scale Λ which depends upon χ. At scales far below Λ the usual renormalization group equations apply with modifications of the high energy boundary conditions. We incorporate these effects by using exact results for large N at scales $\mu \sim \Lambda$ as given in eqs.(20,21), but then use the full RG analysis at lower energies where the higher dimension operators decouple.

The following procedure has been adopted to explore the sensitivity to χ: (i) from $\mu = \Lambda$ to $\mu = \mu^* = \Lambda/5$ we use eq.(20) and eq.(21) directly to evolve Z_H and λ_0; (ii) from $\mu = \mu^*$ to $\mu = m_t$ we use the RG equations. The sensitivity of the low energy predictions is shown in Fig.(4) for the three cases: (1) fermion bubble approximation; (2) ladder QCD; and (3) full standard model. The most sensitive case is that of fermion loop approximation since we see that there is no real nontrivial fixed point to the RG equations in that case. For a wide range of χ the planar QCD and full standard model predictions are very insensitive owing to the nontrivial fixed point for large g_t which is rapidly approached.

Recently Hasenfratz, *et al.* [11] have generalized the Suzuki analysis by including a complete set of higher dimension four fermion interactions. They show that these interactions can cause independent, finite shifts to the values of Z_H and λ_0. With arbitrarily large coefficients of the higher derivative interactions they claim that any physical prediction for m_{top} and m_{Higgs} can be obtained. They conclude that the top condensate theory is unpredictive and that a very light top quark is therefore consistent with the electroweak symmetry breaking coming only from short range interactions of the elementary fermions.

The results of ref.[11] are restricted to the fermion bubble approximation, in which they are true mathematically, but require unphysically large values of the coefficients of the new operators for their conclusions to apply. They require that the finite corrections at the high energy scale dominate the large logarithm arising from the evolution to the weak scale. Moreover, the focusing effects of the infrared fixed points are ignored by considering only the fermion bubble approximation, and these effects will further stabilize the predictions as we have seen previously. As we have demonstrated, the actual results are, in fact, very insensitive to these corrections if the coefficients of the new operators are $O(1)$.

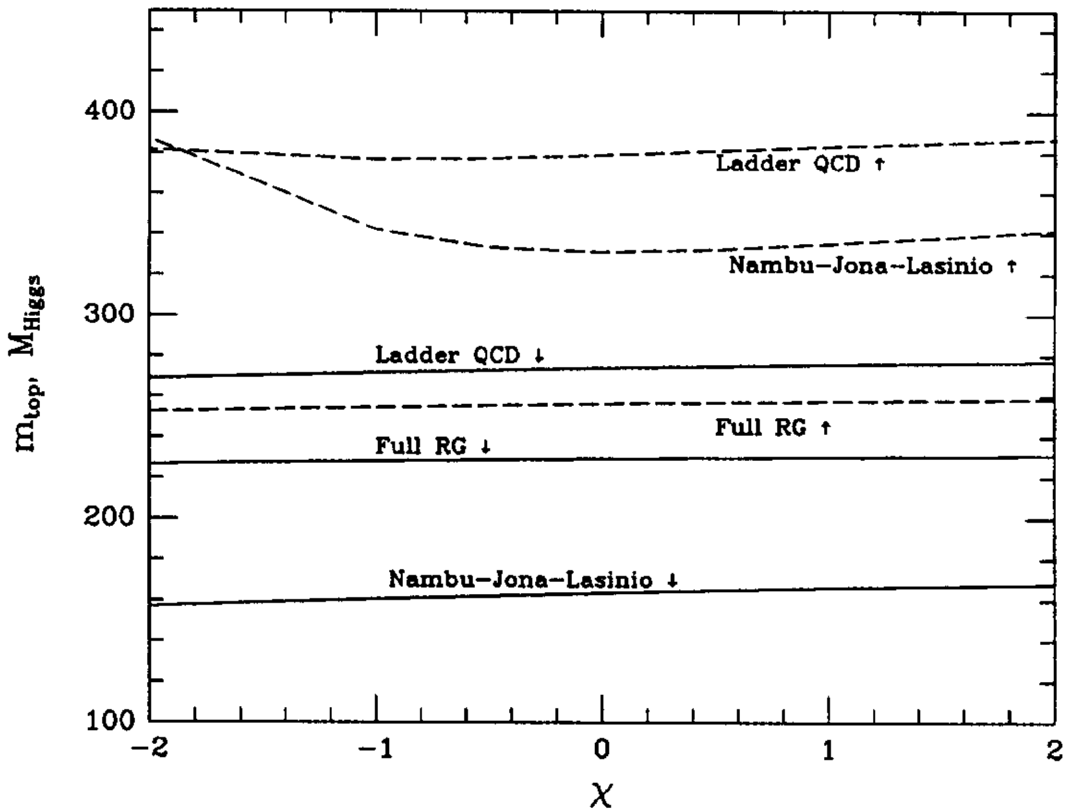

Figure (4): Sensitivity of predicted m_{top} (solid lines) and m_{Higgs} (dashed lines) to $d = 6$ operator coefficient χ.

4. Topcolor: A Gauge Theory that makes a Top Condensate

The ultimate issue of the size of the residual corrections to the leading four-fermion operator resides in the nature of the parent theory, which is valid on scales

$\gg \Lambda$. In a full, realistic theory in which the interactions at the scale Λ are generated dynamically we can hope to compute χ. We turn to this possibility in the next section. In the following we consider one such model, and indeed it is found in ladder approximation that there are residual corrections, but these are very small (we find $\chi \sim 0.1$ rather than $\chi \sim 10$ which is required for any significant impact on the low energy predictions). We refer the reader to ref.[12] for the details of this estimate.

The interaction introduced in eq.(1) is clearly only an effective description of a more primitive theory. A Fierz rearrangement of the interaction leads to:

$$(\overline{\psi}^a_L t_{Ra})_i(\bar{t}_{Rb}\psi^b)^i \to -(\overline{\psi}_{iL}\gamma_\mu\frac{\lambda^A}{2}\psi^i_L)(\bar{t}_R\gamma^\mu\frac{\lambda^A}{2}t_R) + O(1/N) \tag{22}$$

where $N = 3$ is the number of colors. This form strongly suggests a new gauge theory leading to a current–current form of the effective Lagrangian. We further note that: (i) this gauge theory must be broken at a scale of order Λ; (ii) it is strongly coupled at the breaking scale; (iii) it involves the color degrees of freedom of the top quark (or fourth generation fermions) in a manner analogous to QCD. The relevant models will involve the embedding of QCD into some large group G which is sensitive to the flavor structure of the standard model.

Let us construct a minimal version of such a theory.[12] We presently seek a gauge interaction which leads to a term as in eq.(22) but which, like minimal technicolor, will leave the light quarks and leptons massless. A subsequent extension of the theory is required to give masses and mixing angles to light fermions, and we do not address this issue at present. Therefore, consider an extension of the standard model such that at scales $\mu \gg \Lambda$, we have $U(1) \times SU(2)_L \times SU(3)_1 \times SU(3)_2$. We assign the usual light quark and lepton fields to representations under $(SU(2)_L, SU(3)_1, SU(3)_2)$ such that they transform as singlets under the new $SU(3)_2$, as follows:

$$\begin{aligned} (u,\ d)_L; \quad (c,\ s)_L \quad &\to \quad (2,3,1) \\ (\nu_e,\ e)_L; \quad (\nu_\mu,\ \mu)_L; \quad (\nu_\tau,\ \tau)_L \quad &\to \quad (2,1,1) \\ u_R,\ d_R,\ c_R,\ s_R,\ b_R \quad &\to \quad (1,3,1) \\ e_R,\ \mu_R,\ \tau_R,\ (\nu_{iR}) \quad &\to \quad (1,1,1) \end{aligned} \tag{23}$$

while the top quark is a singlet under the first $SU(3)_1$ group:

$$(t,\ b)_L \to (2,1,3); \qquad t_R \to (1,1,3) \tag{24}$$

This assignment is not anomaly free, and we can minimally realize all anomaly cancellations provided we introduce the following electroweak singlet quarks:

$$Q_R \to (1,1,3); \qquad Q_L \to (1,3,1) \tag{25}$$

Both Q_R and Q_L have weak hypercharge $Y = -2/3$, hence electric charge $Q = -1/3$.

Since we wish to break the symmetry $SU(3)_1 \times SU(3)_2 \to SU(3)_c$ at the scale Λ, we introduce a scalar (Higgs) field $\Phi^a_{b'}$ which transforms as $(1, 3, \overline{3})$. By the simplest choice of the Φ potential a VEV develops of the form: $\langle\Phi\rangle = \mathrm{diag}(\Lambda, \Lambda, \Lambda)$. This VEV breaks $SU(3)_1 \times SU(3)_2$ to a massless gauge group $SU(3)_c$ with gluons, A^A_μ and a residual global $SU(3)'$ with degenerate, massive gauge bosons ("colorons") B^A_μ.

Q must be given a large enough Dirac mass, $\gtrsim \Lambda$, so that it does not further influence the dynamical symmetry breaking. We invoke a large Higgs–Yukawa coupling of the Φ field to the combination $\overline{Q}_L Q_R$. Thus, if we take:

$$\kappa \Phi^{b'}_a \overline{Q}^a_L Q_{Rb'} + h.c. \tag{26}$$

then Q receives a mass of $\kappa\Lambda$. A lower bound on κ will be estimated below such that the Q field may be approximated as having decoupled at the scale Λ. It should be noted, however, that with the given the quantum numbers of $\overline{Q}Q$ there is an intriguing possibility that in extensions of this scheme the $\left\langle \overline{Q}Q \right\rangle$ condensate may form dynamically breaking $SU(3)_1 \times SU(3)_2 \to SU(3)_c$, so that an explicit Φ field may not be required. For example, if we assign instead $(c,\ s)_L \to (2, 1, 3)$, we find that anomaly cancellation requires the Q be a triplet with $Y = 0$! Gauging this triplet with yet another $SU(3)_3$ allows a QCD–like chiral condensate of the form $\left\langle \overline{Q}Q \right\rangle$ which is $(1, \overline{3}, 3)$, and the symmetry breaks as described here. This model leads to a low energy two–Higgs doublet scheme.

Returning to the simple example, let the coupling constants of $SU(3)_1 \times SU(3)_2$ be respectively h_1 and h_2. Then the gluon (A^A_μ) and coloron (B^A_μ) fields are defined by

$$\begin{aligned} A^A_{1\mu} &= \cos\theta A^A_\mu - \sin\theta B^A_\mu \\ A^A_{2\mu} &= \sin\theta A^A_\mu + \cos\theta B^A_\mu \end{aligned} \tag{27}$$

where:

$$h_1 \cos\theta = g_3; \qquad h_2 \sin\theta = g_3; \tag{28}$$

and thus:

$$\tan\theta = h_1/h_2; \qquad \frac{1}{g_3^2} = \frac{1}{h_1^2} + \frac{1}{h_2^2} \tag{29}$$

where g_3 is the QCD coupling constant at Λ. In what follows we envision $h_2 > h_1$ and thus $\tan\theta < 1$ to select the top quark direction for condensation. The mass of the degenerate octet of colorons is given by:

$$M_B = \left(\sqrt{h_1^2 + h_2^2}\right)\Lambda = \left(\frac{2g_3}{\sin 2\theta}\right)\Lambda \tag{30}$$

The $SU(3)_c$ current will be the usual QCD current for all quarks while the $SU(3)'$ current (multiplied by its coupling strength) takes the form:

$$\begin{aligned} h_\mu^A &= g_3 \cot\theta \left(\bar{t}\gamma_\mu \frac{\lambda^A}{2} t + \bar{b}_L \gamma_\mu \frac{\lambda^A}{2} b_L + \overline{Q}_R \gamma_\mu \frac{\lambda^A}{2} Q_R\right) \\ &+ g_3 \tan\theta \left(\bar{b}_R \gamma_\mu \frac{\lambda^A}{2} b_R + \overline{Q}_L \gamma_\mu \frac{\lambda^A}{2} Q_L + \sum_i \bar{q}_i \gamma_\mu \frac{\lambda^A}{2} q_i\right) \end{aligned} \tag{31}$$

where the sum extends over all first and second generation quarks. If $h_2 \gg h_1$ the dominant coloron mediated interaction takes the form of eq.(22) provided we identify:

$$\frac{g^2}{\Lambda^2} = \frac{g_3^2 \cot^2\theta}{M_B^2} = \frac{\cos^2\theta}{\Lambda^2} \tag{32}$$

Let is now ask what condition on θ implies dynamical symmetry breaking through the formation of a top condensate. The scale at which the four fermion interaction softens to a gauge boson exchange is given by the mass of the coloron M_B, and we may treat the effective interaction as a four–fermion form at all scales $\mu \ll M_B$. Therefore, in the large–N approximation the gap equation can be written for the spontaneous formation of the top–condensate with a momentum cut–off taken to be $\sim M_B$[5]:

$$m_t = m_t \frac{g_3^2 N \cot^2\theta}{8\pi^2 M_B^2}\left[M_B^2 - m_t^2 \log(M_B^2/m_t^2)\right] \tag{33}$$

and the existence of the condensate implies:

$$\frac{g_3^2 N \cot^2\theta}{8\pi^2} > 1 \qquad \text{or} \qquad \frac{N}{2\pi}\alpha_3(M_B)\cot^2\theta \geq 1 \tag{34}$$

where $\alpha_3 = g_3^2/4\pi$.

On scales below the M_B we expect that the analysis of ref.[5] holds. If $M_B \gg M_W$ then to have an acceptable top mass we must fine–tune θ so that $\frac{N}{2\pi}\alpha_3(M_B)\cot^2\theta \approx 1$ to a high precision. It is also crucial that the spectator Q be sufficiently heavy so that a $\overline{\psi}Q$ condensate *does not form* (the custodial $SU(2)_R$ leads to problems with extra unwanted Goldstone bosons and may ultimately break $U(1)_{EM}$). For a heavy fermion in the gap loop a sufficient condition that no breaking occur in this channel is:

$$\kappa > m_t/(M_B \log(M_B/m_t)) \tag{35}$$

provided the mixing angle is fine–tuned to produce the low mass top condensate. Essentially this condition insures that Q decouples and the associated quadratic divergence becomes $\Lambda^2 - M_Q^2$, and the interaction has insufficient strength to drive the condensate.

We note that χ has been estimated in this scheme and, contrary to the general arguments of Hasenfratz *et al.* , it is found to be small, of order 0.1. For lack of space we must refer the reader to ref.[12], and the Appendix, for a discussion of the size of χ in these models. The approximation of the topcolor dynamics by an NJL model is certainly not established, and much work is required to illuminate the full dynamics. Since NJL implies a second order phase transition, one might study $\langle \bar{t}t \rangle$ as the coupling constant $\cos\theta$ is varied for fixed M_B. If lattice studies can illuminate the behavior it would quite interesting, since we otherwise have no completely reliable methods.

In related works, possible horizontal interactions have been considered by T. K. Kuo, *et al.* , and a $U(1)$ version of this scheme has been developed by R. Bonisch, and independently by M. Lindner and D. Ross.[16]. The same estimate of χ is expected to hold in these schemes as in ref.[12].

5. Conclusions

The main theoretical ideas we have discussed revolve around the notion that conventional quarks or leptons play a fundamental role in the dynamical symmetry breaking of the electroweak interactions. In particular, this provides in the minimal scheme a raison d'etre for the existence of a heavy top quark with a mass of order the weak scale. The predictions of the minimal scheme are completely robust, and very insensitive to the details of the new pairing interactions. The price we pay in such a scheme is the necessity of fine–tuning, which provides the large $\log(\Lambda^2/m_t^2)$, and which ultimately controls the predictions of the model through the infrared fixed points.

Topcolor is a new idea also. We emphasize that this is *not equivalent to an extended technicolor scheme* for two reasons: (a) there is no unbroken QCD–like subgroup in this scheme, and (b) the topcolor interaction is a broken gauge theory but sufficiently strong to drive the formation of a chiral condensate. I believe that the electroweak symmetry breaking involves a new interaction with these properties, albeit not necessarily topcolor, and as such such contains a new previously unseen dynamics.

The predictions of the minimal scheme of ref.[5] yield a top mass of order 230 GeV, which is large compared to current experimental implications through radiative corrections in the electroweak theory. Near future experiments at CDF and D0 will decide the ultimate fate of the minimal top-mode standard model. Nonetheless, this has compelled us to consider the supersymmetric scheme, which allows $m_t > 140$ GeV, and the fourth generation scheme, which does not predict m_t. Both of these schemes have their advantages and flaws. Primarily, they lack the simplicity of the minimal scheme, but they illustrate the fact that the presence of extra degrees of freedom will generally modify the predictions of the minimal scheme, while the general idea of conventional quark and leptons condensates is preserved. There is much more to be done on the theoretical side in this avenue.

6. Appendix

The loops that yield Z_H in the NJL model have no t–dependence, and depend only upon the kinematic variable s. As such, they refer to a point–like boundstate wavefunction on scales $\mu < \Lambda$. However, in any realistic theory we expect there to be modifications to this simple picture, and in general there will t–dependent amplitudes that contribute to the wave–function, hence giving corrections to the naive NJL result for Z_H. We should tackle this problem in a full large–N Bethe–Salpeter analysis. Presently we will make a reasonable guess as to the order of magnitude of these corrections. Let us consider the simplest contribution to such a correction.

For example, we can view the NJL loops as being softened into coloron exchange box–diagrams as in Fig.(5). An upper limit on the size of the t–dependent effects might to take an extreme assumption for the Higgs wave–function and assign all of the incoming momentum on one leg, routing it through the colorons into the opposite outgoing leg. That, indeed, gives an effect of order the present one. A more sophisticated estimate involves making an s–wave projection of the box–diagram onto the incoming $l = 0$ Higgs wavefunction. We shall carry out such an estimate at present. (This appendix is included for illustrative purposes only with this preprint, and will not appear in the published conference proceedings.)

Consider the coloron box diagram of Fig.(5):

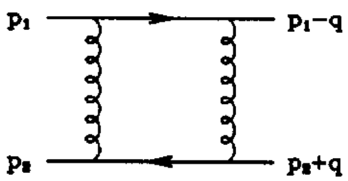

Figure (5): Planar box diagram for computation of topcolor corrections to Z_H

The numerator structure is of the form (the ψ's and q's are external spinors):

$$\beta\bar{\psi}_L \left[(\gamma_\mu(\not{k} + \not{p}_1)\gamma_\nu)\frac{\lambda^A}{2}\frac{\lambda^B}{2}\right]\psi_L\, \bar{q}_R \left[(\gamma^\mu(\not{p}_2 - \not{k})\gamma^\nu)\frac{\lambda^A}{2}\frac{\lambda^B}{2}\right] q_R \tag{36}$$

where

$$\beta = g^4 \cot^2\theta \tag{37}$$

which can be rearranged to the form:

$$-\frac{\beta N}{2}\,\mathrm{Tr}\,[(\not k+\not p_1)(\not k-\not p_2)]\;\bar\psi_L\gamma_\mu\frac{\lambda^A}{2}\psi_L\;\bar q_R\gamma^\mu\frac{\lambda^A}{2}q_R \tag{38}$$

The overall coefficient here is the leading large-N part; the crossed box diagram is subleading. Hence, we compute the leading-N contribution as:

$$Box=-\frac{\beta N}{2}\int\frac{d^4k}{(2\pi)^4}\frac{\mathrm{Tr}[(\not k+\not p_1)(\not k-\not p_2)]}{((k+p_1)^2-m_t^2)((k-p_2)^2-m_t^2)((k+q)^2-M_B^2)(k^2-M_B^2)} \tag{39}$$

By comparison, in the NJL model we have the analogous result:

$$Loop=-\frac{\beta N}{2M_B^4}\int\frac{d^4k}{(2\pi)^4}\frac{\mathrm{Tr}[(\not k+\not p_1)(\not k-\not p_2)]}{((k+p_1)^2-m_t^2)((k-p_2)^2-m_t^2)} \tag{40}$$

which is obtained by pinching out the heavy gluon lines. The external lines will be taken to be massless:

$$p_1^2=p_2^2=0;\qquad q^2-2q\cdot p_1=0;\qquad q^2+2q\cdot p_2=0;\qquad q\cdot(p_1+p_2)=0. \tag{41}$$

and thus:

$$s=2p_1\cdot p_2;\qquad t=q^2 \tag{42}$$

Therefore the box becomes:

$$=-2\beta N\int\frac{d^4k}{(2\pi)^4}\frac{k^2+k\cdot(p_1-p_2)-p_1\cdot p_2}{((k+p_1)^2-m_t^2)((k-p_2)^2-m_t^2)((k+q)^2-M_B^2)(k^2-M_B^2)} \tag{43}$$

Let us now collect the terms associated with the denominators containing m_t. Define a shifted loop momentum:

$$l=k+xp_1-(1-x)p_2;\qquad\text{and}\qquad\Delta^2\equiv m_t^2-x(1-x)s \tag{44}$$

so we have:

$$\begin{aligned}Box\;=\;&-2\beta N\int_0^1 dx\int\frac{d^4l}{(2\pi)^4}\frac{l^2-x(1-x)s}{(l^2-\Delta^2)^2(l^2-M_B^2)^2}\\&\times\left[1-\frac{(q+(1-x)p_2-xp_1)^2}{(l^2-M_B^2)}-\frac{(-xp_1+(1-x)p_2)^2}{(l^2-M_B^2)}\right.\\&+\frac{(2l\cdot(q+(1-x)p_2-xp_1))^2+(2l\cdot((1-x)p_2-xp_1))^2}{(l^2-M_B^2)^2}\\&\left.+\frac{(2l\cdot(q+(1-x)p_2-xp_1))(2l\cdot((1-x)p_2-xp_1))}{(l^2-M_B^2)^2}\right]\end{aligned} \tag{45}$$

where we have expanded to leading order in powers of $1/M_B^2$ and dropped terms odd in l. We now collect terms and we replace $l_\mu l_\nu \to g_{\mu\nu} l^2/4$:

$$Box = -2N \int_0^1 dx \int \frac{d^4k}{(2\pi)^4} \frac{l^2 - x(1-x)s}{(l^2-\Delta^2)^2(l^2-M_B^2)^2} \times \left[1 + \frac{2x(1-x)s}{(l^2-M_B^2)} - \frac{l^2(q^2+6x(1-x)s)}{2(l^2-M_B^2)^2}\right] \tag{46}$$

Wick–rotating to a Euclidean momentum entails:

$$Box = \frac{i\beta N}{8\pi^2} \int_0^1 dx \int du\, u\, \frac{u + x(1-x)s}{(u+\Delta^2)^2(u+M_B^2)^2} \times \left[1 - \frac{2x(1-x)s}{(u+M_B^2)} + \frac{u(t+6x(1-x)s)}{2(u+M_B^2)^2}\right] \tag{47}$$

We now project this amplitude onto the s–wave component. First, the scattering angle is given by:

$$\cos\theta = 1 + \frac{2t}{s}; \qquad d\theta = \frac{-dt}{\sqrt{-ts-t^2}} \tag{48}$$

and the $l = 0$ partial wave is projected by the integral:

$$\langle F \rangle = \frac{1}{\pi}\int_0^\pi d\theta\, F(\cos\theta) = \frac{-1}{\pi}\int_0^{-s} \frac{dt}{\sqrt{-ts-t^2}}\, F(1+2t/s) \tag{49}$$

Hence:

$$Box|_{l=0} = \frac{iN}{8\pi^2}\int_0^1 dx \int_0^\infty du\, u\, \frac{u + x(1-x)s}{(u+\Delta^2)^2(u+M_B^2)^2} \times \left[1 - \frac{2x(1-x)s}{(u+M_B^2)} + \frac{us(12x(1-x)-1)}{4(u+M_B^2)^2}\right] \tag{50}$$

We now perform the integrals yielding the results for the leading terms in the $s/M_B^2 << 1$ limit:

$$Box|_{l=0} = \frac{iN}{8\pi^2}\int_0^1 dx \left[\frac{1}{M_B^2} + O(m_t^2/M_B^4) + + \frac{s}{M_B^4}\left\{x(1-x)(3\ln(M_B^2/\Delta^2) - 6 + 1/2) - 1/24\right\}\right] \tag{51}$$

We perform the x integrals to obtain in the limit $s \gg m_t^2$:

$$
\begin{aligned}
Box|_{t=0} &= \frac{i\beta N}{8\pi^2 M_B^4}\left[M_B^2 + s\left\{\frac{1}{2}\ln(M_B^2/(-s)) + 5/6 - 1 + 1/12 - 1/24\right\}\right] \\
&= \frac{i\beta N}{8\pi^2 M_B^4}\left[M_B^2 + s\left\{\frac{1}{2}\ln(M_B^2/\mu^2) - 1/8\right\}\right]
\end{aligned}
\tag{52}
$$

where we identify $s = -\mu^2$. By comparison the NJL loop is given, with a cut–off of Λ:

$$
Loop = \frac{i\beta N}{8\pi^2 M_B^4}\int_0^1 dx \left[\Lambda^2 + x(1-x)s(3\ln(\Lambda^2/\Delta^2) - 2)\right] \tag{53}
$$

Therefore, we infer the induced wave–function normalization constants for the Higgs boson:

$$
Z_{H\ box} = \frac{N}{16\pi^2}\left\{\ln(M_B^2/\mu^2) - 1/4\right\} \tag{54}
$$

$$
Z_{H\ NJL} = \frac{N}{16\pi^2}\left\{\ln(M_B^2/\mu^2) + 1\right\} \tag{55}
$$

Typically the block–spin renormalization group has been used to obtain Z_H:

$$
Z_{H\ RG} = \frac{N}{16\pi^2}\ln(M_B^2/\mu^2) \tag{56}
$$

We see from the comparison that we must choose $\Lambda^2 = M_B^2$. The terms proportional to s then define the composite Higgs boson wave–function renormalization constant. There is a small constant difference between the NJL loop result and the box diagram. We thus find:

$$
Z_{H\ box} = Z_{H\ RG} - \left(\frac{1}{4}\right)\frac{N}{16\pi^2} \tag{57}
$$

This is a small correction, corresponding to $\chi \approx 1/8$ in eq.(17) and eq.(20).

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